ELECTRON DENSITY POWER SPECTRUM IN THE LOCAL INTERSTELLAR MEDIUM

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February 7, 1994

submitted to *The Astrophysical Journal*

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ABSTRACT

Interstellar scintillation (1SS), fluctuations in the amplitude and phase of radio waves caused by scattering in the interstellar medium, is important as a diagnostic of interstellar plasma turbulence. ISS is also of interest because it is noise for other radio astronomical observations. As a remote sensing tool, 1SS is used to diagnose the plasma turbulence in the interstellar medium (ISM). However, where 1SS acts as a noise source in other observations, the plasma physics of the medium is only of secondary interest. The unifying concern of both studies is the power spectrum of the interstellar electron density.

In this paper, wc usc ISS observations taken through the nearby ($\lesssim$ 1 kpc) ISM to infer the electron density power spectrum of the diffuse interstellar turbulence. From measurements of angular broadening of pulsars and extragalactic sources, decorrelation bandwidth of pulsars, refractive steering of features in pulsar dynamic spectra, dispersion measure fluctuations of pulsars, and refractive scintillation index measurements, wc construct a composite structure function that is approximately powerlaw over $\approx$ 2 X 10^6 m to 10^{13} m. The data are consistent with the structure function having, a logarithmic slope versus baseline less than 2; thus there is a meaningful connection between scales in the radiowave fluctuation field and the scales in the electron density field causing the scattering. The data give an upper limit to the inner scale, Q_o, $\lesssim$ 5 X 107 m and are consistent with much smaller values. Wc construct a composite electron density spectrum that is approximately power law over at least the $\approx$5 decade wavenumber range -1 0^{-8} m^{-1} to 10^{-13} m^{-1} and which may extend to even higher wavenumbers. The average spectral index over this wavenumber range is $\approx$3.7, very close to the value expected for a Kolmogorov process. The outer scale size, L_0, must be $\gtrsim 10^{13}$ m (determine.d from dispersion measure fluctuations). When the 1SS data arc combined with measurements of differential Faraday rotation angle, interstellar cloud densities, and gradients in the average electron density, constraints can be put on the spectrum at much smaller wavenumbers. The composite. spectrum is consistent with a Kolmogorov-like power law over a huge range (12 decades) of spatial wavenumber with an inferred outer scale, $L_0 \gtrsim 10^{18}$ m. This power law subrangc--cxprcssed as ratio of outer to inner scales--is comparable to or larger than that of other naturally-occurring turbulent fluids, such as the oceans or the solar wind. We outline some of the theories for generating and maintaining such a spectrum over this huge wavenumber range.

Subject Headings: Interstellar Medium, Turbulence, Radio Astronomy

I. INTRODUCTION

in the late 1960s and early 1970s, pulsar signals were found to exhibit scintillation caused by interstellar plasma irregularities. The first phenomena to be observed showed the existence of electron density fluctuation power in the interstellar medium (ISM) on scales -109 m (Scheuer 1968; Salpeter 1969; Rickett 1970; Sutton 197 1). Subsequent observations and theoretical arguments suggested the idea of a much wider range of scale sizes in the interstellar medium. Lee and Jokipii (1977), for example, proposed that the $\sim 10^9$ m scale structures could be linked by way of a Kolmogorov-like spectrum with interstellar clouds (scales - 10^{17} m). Armstrong, Cordes, and Rickett (1981) reinforced that idea, summarizing more extensive radio scintillation measurements which gave spectral estimates consistent with the power law idea over several decades in wavenumber. At that time the interstellar scintillation phenomena studied were primarily *diffractive* processes, caused by fluctuation power on scales near the "field coherence scale" (the scale over which the root-mean-square phase difference is 1 radian). More recently, astronomers have also been able to study *refractive* processes caused by irregularities with scales of order the so-called scattering disk scale, which is typically 4 or more orders of magnitude larger than the coherence scale. New observations now allow us to study the power density at the associated lower wavenumbers.

Despite these observational advances, the mathematical form of the ISM density power spectrum and the underlying physics associated with the irregular interstellar plasma continue to be debated. Here we summarize the observational evidence for a power law spectrum of density irregularities in the nearby ($\lesssim$1kpc) interstellar medium. Our approach is model fitting: we assume that the spectrum can be approximated as a power law (usually a Kolmogorov power law) around the wavenumbers that dominate a particular observation. We then determine the spectral density level at those wavenumbers that is required to be consistent with the observation. We combine the inferred spectral levels to produce a composite spectrum over a very wide wavenumber range. We then ask if the aggregate spectrum thus determined is self-consistent, i.e. consistent with a unique power law.

We consider here only the relatively nearby ISM. As has been repeatedly demonstrated (i.e. Cordes, Weisberg and Boriakov 1985; Fey, Spangler, and Mutel 1989; Cordes *et al.* 1991) lines of sight that traverse a kiloparsec or more through the galactic plane are likely to encounter a region producing "enhanced" scatter-il~g. These turbulent clumps arc referred to as "Type B" by Cordes, Weisberg, and Boriakov (1985), whereas their "Type A" component refers to the diffuse, more uniformly distributed component of the turbulence. In a more recent study, Taylor and Cordes (1993) have constructed a more detailed model including enhanced turbulence in the spiral arms. In addition to the diffuse and "enhanced" scattering regions, there are also occasional extreme scattering events (Fiedler *et al.* 1987; Cognard *et al.* 1993) and regions of "depleted" scattering (Phillips and Clegg 1992). It is clear that the electron density field cannot be formally statistically homogeneous on scales much larger than a kiloparsec and indeed the density field may be only approximately homogeneous even within 1 kiloparsec of the Earth (we return to the non-homogeneity issue in Section IV). Within the framework of an assumed unique spectrum for the background turbulence in the ISM, however, our goal is to constrain that spectrum for the diffuse component using observations taken within 1 kiloparsec of the Earth.

'l'his background spectrum is interesting for two reasons. First, the power spectrum is of theoretical interest in the plasma physics of the ISM rtnd in cosmic ray transport studies (c. g., Jokipii 1974, 1988; Cesarsky 1980, McKee and Ostriker 1977; Higdon 1984, 1986). Particularly important in a theoretical explanation of the, irregularities is the steepness of their spectrum. Power law spectra with logarithmic slopes steeper than or equal to 4, which can be due to deterministic structures, have been proposed to explain some of the observations (c. g., Blandford and Narayan 1985). The physical interpretation of such steep spectra is very different from that usually associated with a Kolmogorov-like power law spectrum. (Conventionally, a Kolmogorov spectrum is taken as evidence for turbulent density, velocity and magnetic field fluctuations, with a dynamical energy exchange between irregularities of different scales. Spectra with logarithmic slopes of 4 or more avoid this implied energy exchange and the associated difficulties of explaining the power source, power sink, and bandwidth,) Second, and independent of the physical mechanism(s) involved, scattering from plasma irregularities can introduce noise in diverse radioastronomical measurements. For example imaging through the plasma irregularities can be corrupted by galactic seeing (Cohen and Cronyn 1974, Goodman and Narayan 1989, Narayan and Goodman 1989). Pulsar intensities, radio frequency spectra, and timing arc substantially modulated by 1SS (Cordes, Pidwerbetsky and Lovelace 1986; Romani, Narayan and Blandford 1986) Extragalactic radio sources, too, exhibit intensity variation due to 1SS (Rickett 1986;

Dennison *et al.* 1987; Spangler *et al.* 1993). in searches for very-lc)w-frc(ltlct~cy gravitational radiation, 1SS is a major source of noise which can limit the ultimate sensitivity (c. g., Bertotti, Carr, and Rees 1983; Romani and Taylor 1983; Armstrong 1984; Stinebring *et al.* 1990). We return to the effects of ISM turbulence on searches for VLF gravitational wave backgrounds in the Appendix.

The outline of the paper is as follows. In Section 11 wc establish notation and conventions. Section III summarizes all relevant published data of which we arc aware in terms of the lSM power spectrum and phase structure function. Section IV is a discussion of the composite spectrum inferred from the observations. Section V discusses the physical generation and maintenance of irregularities having a spectrum spanning this huge wavenumber range. Section VI summarizes our results and contains our conclusions.

II. NOTATION AND CONVENTIONS

The theory of interstellar scintillation has been reviewed by Rickett (1977, 1990--henceforth R77 and R90) and Narayan (1992). In Section 111, wc give the reference and equation number used in the interpretation of the observations discussed below. We adopt the Fourier transform convention of R77 and R90. The power spectrum of electron density irregularities, P_{3N}, is the three-dimensional Fourier transform of the electron density autocovariance function (R77, equation 6). The spectrum is taken here to be isotropic, a function of $q = (q_x^2 + q_y^2 + q_z^2)^{1/2}$, where the three dimensional wavevector is (q_x, q_y, q_z). As an *Ansatz*, suggested by previous studies of interstellar and interplanetary turbulence, wc model observation in terms of a powerlaw spectrum with inner and outer scales:

$$P_{3N}(q) = C_N^2\,[q^2 + L_o^{-2}]^{-\beta/2} \exp[-q^2 \ell_0^2/2] \approx C_N^2\, q^{-\beta} \quad \text{for } L_o^{-1} < q < \ell_0^{-1} \qquad (1)$$

Here ℓ_0 is the inner scale and Lois the outer scale. When approximated as an isotropic power law over some range of wavenumber, the spectral index is β and the spectral level is given by the structure coefficient C_N^2. (The Kolmogorov case--β= 11/3--is not the only interesting case for density or magnetic field fluctuations; see Kraichnan (1965).)

A quantity that is more immediately related to some observations is the integrated phase structure function, $D_\phi(r)$. This is the mean square difference in the geometrical optics phase between neighboring straight line paths from a point source to the observer, at a separation of r = $(x^2+y^2)^{1/2}$ in the plane transverse to the propagation direction, z. P_{3N} and D_ϕ arc related by an integral transform (R90):

$$D_\phi(r) = \int_0^z dz' \, 8\pi^2 \lambda^2 r_e^2 \int_0^\infty q \, dq \, [1-J_0(r q z'/z)] \; P_{3N} (q_x,q_y,q_z=0) \qquad (2)$$

where λ is the radio wavelength and r_e is the classical electron radius (2.82 x 10^{-15} m). If the spectrum is an isotropic power law with $\beta < 4$, then the structure function is also power law:

$$D_\phi(r) = 8n2re212 \; C_N^2 \, z \, f(\alpha) \, r^\alpha (\alpha+1)^{-1} \sim (r/b_{coh})^\alpha \text{ for } \ell_0 << r << L0 \qquad (3)$$

Here $\alpha = \beta$-2 and $f(\alpha)$ involves various Gamma functions; $f(\alpha)$ is numerically equal to 1.12 for α=5/3 (Kolmogorov turbulence). With $\beta < 4$ there is a meaningful mapping between wavenumber, q, and spatial scale, r: q = 1/r. For steeper spectra the structure function has a square-law form and there is no simple wavenumber-to-scale mapping. in this paper wc convert from spatial scale to wavenumber according to q = l/r, i.e. assuming $\alpha < 2$, and discuss this assumption in Section IV.

D_ϕ is used to define the coherence scale, b_{coh}, for the electromagnetic field: $D_\phi(b_{coh}) = 1$ rad^2. D_ϕ scales exactly as (radio wavelength)^2 and as (propagation distance)] for homogeneous turbulence. Wc usc this property to scale measured or inferred phase structure functions to a standard distance (1kpc) and a standard radio frequency (1 GHz), denoting this scaled phase structure function as $D_{\phi,1GHz/1kpc}$.

Some of the observations summarized in the next section are of intensity variations. Spatial structure in the intensity field is in general only indirectly related to spatial structure of the irregularities in the medium. An important special case is when the scintillation is "weak"; in weak scintillation the first Born approximation applies and the amplitude variations are a linearly filtered version of the phase perturbations imposed by the medium. The filter passes scales smaller that the Fresnel scale $(z\lambda/2\pi)^{1/2}$, where z is the effective propagation distance through the ISM and λ is the radio wavelength. The resulting rms intensity, normalized by the mean intensity, is the Born scintillation index, m_B. A condition for validity of the first Born approximation is $m_B^2 << 1$; when this is true the scintillation is weak and m_B gives a measure of the medium perturbations on the Fresnel scale. When $m_B^2 > 1$ the scintillation is strong and the structure of the intensity field splits into two component% These are referred to as diffractive interstellar scintillation (DISS) on scales near b_{coh} and refractive interstellar scintillation (RISS) on scales near $R_S = z\,\theta_S$, the radius of the "scattering disk". Here θ_S $(= \lambda/2\pi b_{coh})$ is the angular extent of the radiation received by an observer and R_S is the largest scale in the medium that can be detected by observing the intensity of the received field. θ_S is the resolution limit in conventional imaging and $z\theta_S{}^2/2c$ represents the shortest time structure resolvable in pulse measurements.

In contrast with strong intensity scintillation, dispersion measure (DM) fluctuations are directly connected to the electron density structure function (e.g., Lovelace 1970) and are easily interpreted. These DM observations provide an important and very direct measure of the phase structure function, and thus ISM irregularity power, on scales much larger than the Fresnel scale.

111. OBSERVATIONS

In this section wc present radio scattering and other observations and interpret them in terms of phase structure function (scaled to a standard radio frequency of 1 GHz and a standard propagation distance of 1 kpc) and the power spectrum of density in the ISM.

Diffractive Interstellar Scintillation (DISS) index

At 2.7 GHz, the diffractive scintillation index, m_{DISS}, is approximately 0.5 for a propagation distance $z = 3.5 \times 10^{18}$ m (Backer 1975; R77). Since m_{DISS} is less than unity, the scintillation is weak and an estimate of the ISM power level can be made near the Fresnel wavenumber (weighted along the line of sight). Using R77, equation (38), with β = 11/3 wc get $P_{3N}(2 \times 10^{-8}\ m^{-1}) = 8.4 \times 10^{24}\ m^{-3}$. The associated phase structure function value is $D_{\phi,1GHz/1kpc}(5 \times 10^{7}\ m) = 6.5\ rad^2$. These values arc plotted as squares on Figures 1 and 2.

At meter wavelengths and for distances to local pulsars, m_{DISS} is approximately unity, indicating strong scintillation. This puts a lower limit on P_{3N} near the meter wavelength 1SS Fresnel wavenumber (provided the spectrum has $\beta < 4$) since the spectral level near the Fresnel wavenumber must be such that $m_B^2 > 1$. Armstrong and Rickett (1981) observed PSRs 0329+54 and 1133+16 at 340-408 MHz and measured $m_{DISS} = 1$ (to within a few percent). The best limit is from 1133+16; from R77, equation (38), and using a line-of-sight weighted Fresnel scale, gives $P_{3N} > 4.4 \times 10^{24}\ m^{-3}$ near $q = 6 \times 10^{-9}\ m^{-1}$ (or $D_{\phi,1GHz/1kpc}(1.7 \times 10^{8}\ m) > 0.4\ rad^2$). These bounds arc plotted as the indicated lower limit symbol in Figures 1 and 2 and provide useful constraints independent of interpretational difficulties relating to an inner scale.

Decorrelation Bandwidth of DISS

An estimate of the spectral level near $1/b_{coh}$ can be obtained from observations of the decorrelation bandwidth made at meter wavelengths. From Cordes *et al.* (1991) the structure constant C_N^2 is related to the decorrelation bandwidth of pulsars for a Kolmogorov-spectrum scattering medium via:

$$C_N^2 = 292\,(2\,\pi B_{Hz})^{-5/6}\ \nu_{GHz}{}^{11/6}\, z_{kpc}{}^{-11/6}\ \ ,1,-20/3 \quad (4)$$

where B_{Hz} is the decorrelation bandwidth in Hz, ν_{GHz} is the radio frequency in GHz, and z_{kpc} is the propagation distance in kpc. The coherence scale is determined from equation (3). The radio wavelength scaling of decorrelation bandwidth and the shape of the autocorrelation function of 1SS in radio frequency arc consistent with $\beta = 11/3$ near this scale (Cordes, Weisberg, and Boriakoff 1985; Armstrong and Rickett 198 1), justifying our usc of formulas which assume a locally Kolmogorov spectrum. (Wc return to this point in Section IV, however, in connection with constraints on an inner scale.) Cordes, Weisberg, and Boriakoff (1985) made extensive observations of decorrelation bandwidth and summarized others from the literature. Restricting to pulsars with dispersion measure < 30 cm^{-3}pc and associating the observable with the wavenumber b_{coh}^{-1}, wc obtain the 111 estimates of P_{3N} and $D_{\phi,1GHz/1kpc}$ (on 24 pulsars, each typically observed at several radio frequencies), plotted as small open circles in Figures 1 and 2.

Meter Wavelength Angular Broadening

Low radio frequencies arc typically required (o scc a measurable angular broadening effect over the relatively short propagation distances considered here. VLBI observations of PSRS 0834+06, 0950+08, 1133+16 and 1919+21 at radio frequencies in the range 74-196 MHz give 1SS scattering diameters in the range 0.05-().07 arcseconds on baselines b $\approx 2.6 \times 10^6$ meters (Resch 1974; Vandenberg 1974). Wc associate these observations with wavenumber q = b-*. Using the Kolmogorov formulas the inferred values of C_N^2 arc in the range 4×10^{-4} to 2×10^{-3} $nl^{-6.67}$, giving $P_{3N}(3.8 \times 10^{-7}\, m^{-1})$ in the range (1.5 to 7) X $10^{20}\, m^{-3}$ and $D_{\phi,1GHz/1kpc}(2.6 \times 10^6\, m)$ between 0.04 and 0.17 rad^2. More recent 327 MHz VLBI observations of pulsars have been reported by Gwinn *et al.* (1993). Of the three "local" pulsars in their sample (Vela is excluded due to known anomolously large scattering near the pulsar) only PSR 1919+21 shows any evidence for scatter broadening. The visibility is ≈0.9 on a projected baseline of ≈6.4 X 1 0^6 m. This gives $D_{\phi,1GHz/1kpc} \approx 0.07$ rad^2 for this baseline and $P_{3N}(1.6$ X 10^{-7} m-])= 1.4X 10^{21} m^{-3}.

Angular diameters of extragalactic radio sources in the survey of Readhead and Hewish (1974) have been interpreted in terms of 1SS angular broadening by Duffett-Smith and Readhead (1976) and Hajivassiliou and Duffett-Smith (1990). Using equation (19) of R77 (scattering size= 0.1" at 81.5 MHz and typical propagation distance of 500 pc through the galaxy) gives C_N^2 -2.6 X 1 $^{0-4,11-6.67}$ We associate this with an interferometer baseline ≈3.2 X 10^6 m having a resolution of ≈0.1 arcsec at 81.5 MHz, thus P_{3N} (3.1 X 10^{-7} m^{-1}) ≈2 X 102° m^{-3} and $D_{\phi,1GHz/1kpc}(3.2 \times 10^6$ m) ≈ 0.03 rad^2. Spectral and structure function level estimates determined from these VLBI and IPS observations are shown in Figures 1 and 2 as filled circles.

Frequency Drift in Pulsar Dynamic Spectra

Meter wavelength dynamic spectra of pulsars show strong modulation in frequency and time duc to diffractive 1SS. Often, in addition to apparently random variations in frequency and time, organized structures arc observed in the spectra (see R90 for a discussion). Onc prominent structure is drifting bands, features that organize along sloping lines in the dynamic spectra. These bands arc interpreted as duc to refractive steering of the DISS pattern by much larger scale structures that are sensibly static over a DISS integration time (Shishov 1974; R77; Hewish 1980; R90). The basic idea is that the refractive steering angle can be approximated by&= $(\lambda/2\pi)\nabla\phi$, with $\nabla\phi$ averaged over the scattering disk scale. θ_r depends on radio wavelength since the geometrical optics phase (ϕ) is proportional to λ^1 for a cold plasma. Refractive steering hence displaces the DISS pattern by an amount $\approx z\,\theta_r/2$, where z is the distance to the source. Thus if distance is related to time via a pattern speed, v_{pat}, features appear as a band in (frequency, time). The slope at a center frequency f is given by (Hewish 1980):

$$\frac{dt}{df} \approx \frac{z\,\theta_r}{v_{pat}\, f} \qquad (5)$$

(Usually measurements of df/dt are reported, but its reciprocal has a zero mean and easier to connect with the theory.) From dt/df, a pulsar distance estimate, and a pattern speed, one obtains θ_r. We interpret this single sample as estimating therms angle of refraction, which in turn can be related to D_ϕ by approximating $|\nabla\phi|$ as $[D_\phi(R_s)]^{1/2}/R_s$, where $R_s = z\,\theta_s$ is the size of the scattering disk. We determine θ_s from decorrelation bandwidth measurements and use pattern velocities measured in the same observation (if possible) or from the literature (e. g., Cordes 1986; Harrison *et al.* 1993). The data shown in the Figures are from Armstrong and Rickett (1981) and Smith and Wright (1985), selected for $z < 1$ kpc. The inferred values of P_{3N} and $D_{\phi,1GHz/1kpc}$ are plotted as large open circles at $q = 1/R_s$ and at $b = R_s$.

Pulsar Dispersion Measure Fluctuations

Electron density fluctuations on scales up to -5 X 10^{12} m have been observed directly through fluctuations in dispersion measure (DM). Phillips and Wolszczan (1991) monitored PSRS 0834+06, 0823+26, and 0919+06 for 2 years and constructed the DM structure function for spatial scales in the approximate range 3 X 10^{11} m to 5 X 10^{12} m. Since DM is essentially a straight line integral of the electron density from the source to the observer, it is directly related (Lovelace 1970) to the phase perturbation, ϕ. Phillips and Wolszczan (1991) present their DM structure functions as phase structure functions for a radio frequency of 430 MHz, $D_{\phi430}$. We convert to our standard 1GHz and 1kpc according to $D_{\phi,1GHz/1kpc}(r) = D_{\phi430}(r)\,(0.3\mathrm{m} / 0.7\mathrm{m})^2\,(1/z_{kpc})$, where z_{kpc} is the distance to the pulsar in kiloparsec. To obtain the associated spectral levels we use the powerlaw formula (equation (3) with α determined from the Phillips and Wolszczan (1991) data), to evaluate C_N^2 and hence $P_{3N}(q = 1/r)$. The results are plotted in Figures 1 and 2 as straight lines.

The justification for using powerlaw spectrum formulas to go from $D_{\phi430}$ to P_{3N} is that the phase structure functions measured by Phillips and Wolszczan (1991) are consistent with $\beta \approx 11/3$ powerlaws. In addition, Phillips and Wolszczan (1 991) compared their observed structure function values on large scales with those inferred at much smaller scales from decorrelation bandwidth measurements. This was done for each pulsar individually and thus removed the uncertainties of comparing different lines-of-sight. Their result very strongly indicates that the average spectrum is a near-Kohnogorov power law (indices between 3.77 and 3.87) over about 6 decades in wavenumber for these three lines of sight. The average index connecting the wavenumbers for DISS and the DM fluctuations is plotted as a dotted line in the upper panel of Figure 1; similarly in Figure 2, (Analyses of the DM fluctuations of 1937+21 by Cordes *et al.* (1990), Rickett (1988), Stinebring *et al.* (1990), and Ryba (1991) give results that arc in quantitative agreement with the Phillips and Wolszczan (1991) structure functions. The 1937+21 data arc not included in Figures 1 and 2, however, since the distance to 1937+21 is too large to be included as "local" data.)

Pulsar Timing Noise

Closely related to DM fluctuations arc measurements of the residual time-of-arriva] of pulsar pulses. Hellings and Downs (1983) and Romani and Taylor (1983) presented spectra of pulsar timing noise for several pulsars within 1 kiloparsec of the earth. These observations are certainly dominated by effects other than 1SS, but can be used to construct upper limits for P_{3N} in the wavenumber range $10^{-13.4}$ to $10^{-12.0}$ m^{-1}. Figure 2 of Hellings and Downs (1983) presents the power spectrum of timing noise (expressed there as the power spectrum of fractional frequency fluctuations) in the band $10^{-8.4}$ to 10^{-7} Hz. Using an assumed pattern velocity of 100 km/sec, a distance of 500pc, and equation (2) of Armstrong (1984) (see also the Appendix) which relates power spectrum of frequency fluctuations to P_{3N} under the Kolmogorov spectrum assumption, results in the indicated upper limits for that band.

Stinebring *et al.* (1990) report 1.7 μsec RMS timing noise on PSR 1855+09, based on 66 observations taken over 3.9 years at 0.21 m observing wavelength. Based on their Figure 1, the spectrum of those residuals is approximately white, thus the spectral density of the timing noise has level -10-5 sec^2/Hz in the Fourier band in which they have observations. Converting this to a spectral density of fractional frequency fluctuations at the band center (=1.4 X 10-7 Hz), assuming $'_{pat}$ = 100 km/sec, taking the distance inferred from the dispersion measure, as using equation (2) of Armstrong (1984) gives an upper limit to P_{3N} of about 10^{11} m^{-3} at q = 1.4 X 10^{-12} m^{-1}. This and the associated phase structure function value arc plotted as upper limits in Figures 1 and 2.

Refractive Interstellar Scintillation (RISS)

Pulsar intensity fluctuations on time scales of days to months have been identified as the refractive counterparts of diffractive fluctuations having time scales of minutes to hours. The latter have a scintillation index m_{DISS} near unity and arc caused by irregularities on about the coherence scale. B y contrast, refractive variations are caused by power having spatial scales several orders of magnitude greater, about equal to the scattering disk size, R_s. The refractive scintillation index, m_{RISS}, determined from intensity variations on time scales $\sim R_s/v_{pat}$ is a measure of the power in the density spectrum at wavenumbers near $1/R_s$ for any powerlaw with $\beta < 4$. For steeper spectra, $\beta > 4$, m_{RISS} should saturate near unity. This distinction provides an important constraint on β, since the measured values of m_{RISS} arc near or slightly above those expected for a Kolmogorov spectrum. Kaspi and Stinebring (1 992) find m_{RISS} consistent with $\beta = 11/3$; Gupta *et al* (1993) find somewhat larger values for m_{RISS} but still substantially below unity. Such measurements provide strong evidence that $\beta < 4$ near $q \approx 1/R_s$ and thus rule out models with β just above 4 that had been proposed by Blandford and Narayan (1985). (The special case of $\beta = 4$ requires additional attention which wc postpone to Section IV.)

Proceeding on the basis that $\beta < 4$ wc can usc published values of m_{RISS} for various pulsars to estimate P_{3N} at wavenumbers near $1/R_s$. The scattering disc size is estimated from the pulsar distance and measured decorrelation bandwidths, scaled to the radio frequency of the m_{RISS} observations using Kolmogorov scaling rules; this results in R_s values typically 10^{12}-10^{13} meters. The normalized refract.ivc scintillation variance for a spherical wave (point) source illuminating an extended, approximately Kolmogorov medium of depth z, as a function of the spectral level for irregularities scales near the scattering disc size, $P_{3N}(q = 1/R_s)$, is given by

$$m_{RISS}^2 \approx 0.95\, r_e^2 k473\ R_S^{-6}\, P_{3N}(q = R_s^{-1}) \qquad (6)$$

Wc derived this relation from equation (A2) of Colts *et al.* (1987). in Figures 1 and 2 wc have taken the observations of those pulsars which have estimates of both m_{RISS} and $\Delta\nu_{DISS}$ (wc infer the scattering angle from $\Delta\nu_{DISS}$ and thus the scattering disc size). The m_{RISS} data arc from

Rickett and Lyne (1990), Kaspi and Stinebring (1992), and Gupta *et al. (1993),* selected for $z \lesssim 1$ kpc. These data are plotted as star symbols in Figures 1 and 2.

observations of finite-diameter *extragalactic* sources are also useful, where the relevant scale for refractive intensity fluctuations is now $z\,\theta_{source}$. At high galactic latitudes θ_{source} is typically greater than θ_S, and so the scale probed can be even greater than Rs for a point source having the same propagation distance. We use equation (17) of Coles *et al.* (1987) to interpret the values of θ_{source} and' m_{RISS} for the high-galactic latitude ($b > 30^{o}$) sources observed by Spangler *et al.* (1993). These are shown as boxes in Figures 1 and 2.

Rotation Measure of Extragalactic Radio Sources

Simonetti, Cordes, and Spangler (1984), Simonetti and Cordes (1988), Lazio, Spangler, and Cordes (1990), Clegg *et al.* (1992), and Simonetti (1992) have observed differential Faraday rotation of double radio sources on radio paths through the galactic plane. Lazio *et al.* (1990) constructed the structure function of rotation measure. (RM) and, using reasonable parameters for the scattering volume, inferred the density power spectrum on spatial scales -0.1-70 pc ($\approx 3 \times 10^{15}$ to 2×10^{18} m) Values of P_{3N} consistent with the RM fluctuation measurements are constructed using equation (6) of Lazio *et al.* (1990) and the observed structure function of RM. These are plotted in Figures 1 and 2 as upper limits because the relative contribution of electron density and magnetic field fluctuations is unknown; the indicated limits assume all of the RM fluctuations are due to electron density fluctuations. The region investigated by Lazio *et al.* (1990) is characterized by particularly heavy scattering (Fey, Spangler and Mutel 1989), so that local value of C_N^2 in that region almost certainly greatly exceeds that within 1 kpc of the sun. Accordingly, we regard the data points representing the Lazio *et al. (1990)* measurements in Figures 1 and 2 as very conservative upper limits.

Estimate Near Density Outer Scale

An estimate of the power spectrum level at very low frequencies can be obtained from a fairly general gradient scale argument (Lee and Jokipii 1977; Ishimaru 1978). This argument goes as follows. The density structure function is the mean square of the difference in electron density at a given separation. Near the outer scale, Lo, the structure function value should be comparable with the square of the difference in the *average* density on a separation of Lo: $D_N(L_o)$ - (L. ‘$\mathrm{Pave})^2$. Taking pave ≈ 0.03 cm^{-3} and Lo= 100 pc (Lee and Jokipii 1977; Kaplan 1966), and the gradient scale for pave to be ≈500pc (Readhead and Duffett-Smith 1975; see also Taylor and Cordes 1993) gives $D\rho(L_o)$ -4 X 10^7 m^{-6} or $D_{\phi,1GHz/1kpc}$ 10^{16} rad^2 at separation 100pc. If, as for a Kolmogorov spectrum, $D_\rho(x) = (C_N^2/0.033)\, x^{2/3}$ then C_N^2 -0.033 $L_o^{4/3}\, (\nabla\rho_{ave})^2$ -10-6”2 m 6.67 and $P_{3N}(3.2 \times 10^{-19}\, m^{-1})$ -1061.5 m^{-3}. These arc plotted as open circles in Figures 1 and 2 with ± two orders of magnitude uncertainties reflecting our subjective assessment of the uncertainties in measured quantities and their applicability in this argument. (Note that D_ϕ and P_{3N} should start to “saturate” near the outer scale L_o and L_o^{-1}, respectively; the Faraday rotation upper limits arc consistent with such a transition but arc not in themselves compelling evidence for L_o - 100 pc; scc Section IV.)

Other Indications of Large Scale Structure

Velocity fluctuations of ISM gas and the existence of interstellar clouds suggest that the plasma turbulence spectrum extends to wavenumbers significant] y lower than the -2X $10^{-13}\, m^{-1}$ indicated by the RISS and DM fluctuation observations.

The velocity structure function measured from the motions of stars and neutral gas is strikingly Kolmogorov, $D_V(x) \approx 20\ km^2/sec^2\, (x/70pc)^{2/3}$, over at least the distance scale range 1-70 pc (Kaplan 1966; Larson 1979). To relate the velocity data to the desired electron density structure function, wc need to assume a model of the ISM where the neutral gas velocities trace the plasma velocities on the large scales and usc an analogy with the interplanetary medium. In the McKee and Ostriker (1 977) model of the ISM, cool (- 100K) dense neutral clouds arc surrounded by warm (~10^4 K) gas (partially ionized by stellar UV), and embedded in a hot (~10^6 K), diffuse, fully-ionized coronal phase. At low wavenumbers, fluctuations in the electron column density on typical pulsar distances probably reflect fluctuations in the warm phase, which is thought to be closely associated with the cool neutral phase. In this view, the motion of the neutrals and the low-frequency plasma fluctuations arc coupled. For scales small with respect to those of the overall flow, the fractional density fluctuations in the quiet interplanetary medium arc observed (Neugebauer, Wu, and Hubs 1978) to be proportional to the velocity fluctuations divided by the Alfven speed: $|\delta\rho / \rho_{ave}| = a\,|\delta v/v_A|$ where $a = 10^{0\pm 1}$. Thus $C_N^2 = a^2 C_v^2 (\rho_{ave}/v_A)^2$, where C_v^2 is the structure coefficient for the spectrum of velocity fluctuations. We convert to P_{3N} using $\rho_{ave} = 0.03\ cm^{-3}$ and $v_A = 10$ km/sec. This results in an inferred spectral level and shape approximately consistent with an extension of the higher wavenumber spectra, but now on much lower wavenumbers . Because of the model-dependency and uncertainty in relating these quantities, we show these estimates as a dashed box. The uncertainty in "a" is reflected in a $\pm$ two order of magnitude vertical extent of the box. in the upper panels of figures 1 and 2 wc plot the spectral index of the velocity fluctuations.

The. warm, partially ionized phase of the ISM is predicted to fill a significant fraction of space as shells of partially ionized gas, having typical electron densities -0.2 cm^{-3}, depth of modulation $<\rho^2> / \rho_{ave}^2$ of order unity, and typical radii -2 pc. The existence of interstellar plasma clouds allows a point to be placed on the spectrum at q - 1 /(typical cloud size), whether or not there is any dynamical relationship with irregularities on other scales. If these clouds arc approximated as spheres having Gaussian radial profiles, then the associated spectral density $P_{3N}(\sim 1.5 \times 10^{-17} m^{-1})$ -1058 m^{-3} is consistent with the dashed box obtained from the velocity fluctuation data in Figures 1 and 2; this point is plotted as a diamond-shaped symbol.

IV. DISCUSSION OF THE WAVENUMBER SPECTRUM

The data in Figures 1 and 2 arc most simply interpreted in terms of a pure powerlaw spectrum with $\beta \approx 11/3$ over wavenumbers $10^{-6.3}$ to $10^{-12.7}$ m^{-1} (the range of radio scattering data); when combined with the non-radio data at lower wavenumbers, the data are consistent with a $\beta < 4$ powerlaw over a huge range, down to q - 10-16 m^{-1} or smaller. This interpretation calls for ℓ_0 no larger than about 10^6 m and Lo greater than about 10^{16} m and is consistent with the assumptions wc used to plot the points in Figures 1 and 2, namely that the density spectrum P_{3N} is a power law and that it has spectral exponent $\beta < 4$. We need to examine the basis for these assumptions and ask if other spectral models (e.g. models with larger inner scales) could fit the data comparably well. In this section wc review these assumptions, discuss the internal consistency of the data set, and comment on our fundamental assumption of statistical homogeneity of the local ISM which allows data from different lines-of-sight to be combined in a composite power spectrum.

Exclusion of Steep Spectrum Models

The large cluster of points in Figure 1 near wavenumbers of 10^{-7} m^{-1} establishes the level of P_{3N} but does not directly constrain the exponent β. The observed autocorrelation function of DISS in radio frequency for a fcw pulsars (Armstrong and Rickett 1981) and the scaling of DISS decorrelation bandwidth of PSR 0329+54 with radio frequency (Cordes, Weisberg and Boriakoff 1895) arc consistent with $\beta \approx 11/3$ over perhaps a decade of wavenumber near 10-7 m^{-1}. (Cordes *et al.* (1985) observed the wavelength scaling of decorrelation bandwidth for several other pulsars, which also give scalings consistent with $\beta < 4$, but most arc too distant to qualify as "local".) Blandford and Narayan (1985) noted that there was an alternative interpretation for the wavelength scaling of decorrelation bandwidth with β in the range 4.2 to 4.6. Such values, however, are now ruled out by the observations of m_{RISS}; values of $\beta > 4,()$ would yield m_{RISS} c1osc to or equal to unity, in contradiction with the observations. The other important ncw data set in this context is the DM structure function, which constrains the average β over wavenumbers from =10-7 m^{-1} to $\approx 10^{-13}$ m^{-1}. Phillips and Wolszczan (1991) give a mean exponent for the structure function, α, over this range of 1.83 ± 0.05, where the uncertainty quoted is half the range among 3 local pulsars.

The critical question is whether $\alpha = 2$ is excluded, since for all spectra with $\beta > 4$ the structure function exponent α equals 2. The DM variations combined with measurements near the coherence scale might be marginally consistent with $\alpha = 2$ between $\sim 10^{-7}$ and 10^{-13} m^{-1} but the observation that $m_{RISS} < 1$ is not. Thus wc conclude that $\beta < 4$ for the wavenumber range $\sim 10^{-7}$ and 10^{-13} m^{-1}. Extending this conclusion that $\beta < 4$ down to wavenumbers $\sim 10^{-16}$ m^{-1}, relics on the gradient scale point, the ISM cloud point, the velocity fluctuation region and, importantly, the very conservative nature of the upper limits derived from Faraday rotation variations. The Faraday rotation limits in Figure 1 are formally consistent with a value $\beta = 4.0$ between about 10^{-13} m^{-1} and about 10^{-17} m^{-1}. However, as indicated above, the Faraday rotation data were derived from the Cygnus region, which exhibits turbulence that is substantially enhanced over that in the local 1 kpc. This and the other non-radio-data points in Figure 1 suggests that the true local spectral level is substantial] y below the plotted upper limits, arguing for $\beta < 4$ between 10^{-13} m^{-1} and 10-17 m^{-1}, and consistent with $\beta = 11/3$ down to perhaps as low a few times 10^{-18} m^{-1}. The special case of a $\beta = 4$ spectrum is discussed in the next subsection.

The β = 4 Spectrum

If the interstellar plasma were entirely clumped into clouds with abrupt edges, the density spectrum would show a slope of $\beta = 4$ for the range of wavenumbers between the reciprocal of the typical cloud size and the reciprocal of the thickness of an “abrupt” edge. Lines of sight that pass through any abrupt structure, such as a shock due to an expanding supernova cavity or an assembly of pressure balance structures, will also exhibit a $\beta = 4$ spectrum. The ratio of the shock thickness to the characteristic scale of the region would determine the wavenumber range having $\beta = 4$; this could extend over several decades if the edges of these structures could be made thin enough. Such a spectrum dots not imply a turbulent cascade, since there would be no dynamic interactions between irregularities on different scales.

The scattering theory for this case has not been considered very actively, although its logarithmic structure function has been discussed by Rumsey (1975), Goodman and Narayan (1985), and Dashen *et al. (1993).* In strong scattering with a $\beta = 4$ spectrum, diffractive and refractive components of the field arc produced, as is the case for "conventional" ($\beta < 4$) spectrum models. The logarithmic structure function is asymptotic to $\alpha = 2$ for scales much smaller than the outer scale, but approaches the asymptote only very slowly. As a result it can have "local" slopes between 3.5 and 3.9 over several decades of wavenumber. For example, Figure 2 contains two dashed lines representing a $\beta = 4$ model with outer scale Lo of 10^{13} m and 10^{16} m, normalized vertically so as to pass through the cluster of points near the coherence scale at $\approx 2 \times 10^{7}$ m. Either is in reasonable agreement with all the radiowave scattering points and limits (scales of order a few times 10^{6} to $\sim 10^{13}$ m), but a $\beta = 4$ model with Lo significantly smaller than 10^{16} m would disagree with the non-radiowave points at low spatial frequency.

This $\beta = 4$ model has, however, not yet been developed completely. A comparison is needed with all of the pulsar observations of DISS and RISS, and is beyond the scope of the present paper. We simply note here that the associated m_{RISS} would lie well below unity and is approximately compatible with most of the data for nearby pulsars. Thus $\beta = 4$ in the range 10-7 to 10^{-13} m^{-1} with outer scale appropriatel y taken to get the observed value of α near the RISS scales is not yet ruled out by either the DM structure functions or the 1<1SS data in the lower panel of Figure 2. Such a spectrum would, however, predict α very c1osc to 2 at the DISS scales, apparently in conflict with the wavelength scaling of decorrelation bandwidth and the shape of the DISS autocorrelation in radio frequency (upper panel of Figure 2).

Constraints on the Inner Scale

Returning to the most obvious interpretation of Figures 1 and 2, that there is a very wide wavenumber range with a power law behavior $\beta < 4$, wc now address the question of a possible inner scale cut-off to the spectrum. Such an inner scale would make the spectrum steeper than 4 for wavenumbers above]/Q. and thus the structure function would vary like $(\text{scale})^2$ for scales< ℓ_0. In this discussion wc assume that there is a Kolmogorov range corresponding to $\beta = 11/3$, and ask what evidence wc can present about LO.

Observations of angular broadening, decorrelation bandwidth, DM and RM fluctuations, and refractive steering of DISS arc fundamentally measures of the structure function, rather than the spectrum. Our association of scale with wavenumber (i.e., scale = 1/wavenumber) used to plot those observations on Figure 1 assumed that $\alpha < 2$ (which wc emphasize is consistent with all the available data on the slope of the structure function near the scales sensed by each observable). In contrast with the observables listed above, weak intensity scintillation at centimeter wavelengths is fundamentally sensitive to the spectrum near the Fresnel scale. Thus the point at $q = 2 \times 10^{-8}\ m^{-1}$ in Figure 1 is plotted correctly and gives an immediate, essentially model-indcpcndcnt upper limit ℓ_0: ℓ_0 must bc smaller than about 5×10^7 m. Arc the coherence bandwidth and angular broadening data consistent with such a large value of inner scale?

For an inner scale $\ell_0 \sim 10^8$ m, the spectral model in Figure 1 drops below the $\beta = 11/3$ line for wavenumbers larger than $10^{-8}\ m^{-1}$. This is shown by the curved dotted line, which passes substantially below all the points near $10^{-7}\ m^{-1}$. This at first seems to rule out such large inner scale values. However, we have to examine the theory of DISS under the condition that $\ell_0 > b_{coh}$. The DISS decorrelation bandwidth still provides an estimate of b_{coh}, the associated cluster of points arc thus correctly plotted as D_ϕ in Figure 2. Under the hypothesis that $Q_0 \approx 10^8$ m, a model line would have a slope of 2 for scales smaller than 10^8 m and break to a slope of 11/3 for larger scales. The D_ϕ plot thus seems compatible with inner scale $\approx 10^8$ m. If ℓ_0 were as large as 10^8 m, wc then have to ask where the DISS bandwidth and angular broadening points should bc placed on Figure 1. If $\ell_0 > b_{coh}$ and if $r < Q_0$, neither the mapping $q = 1/r$ nor equation (3) applies and it can bc shown that:

$$D_\phi \approx r^2\, 2\, \pi^2\, r_e^2\, \lambda^2\, z\, \frac{P_{3N}(q = 1/\ell_0)}{(4-\beta)\,(\beta-1)\,\ell_0^4} \qquad (7)$$

The result is given at $q = 1/\ell_0$ to avoid assuming a detailed form for the cutoff beyond this wavenumber. Setting $r = b_{coh}$ gives $D_\phi = 1\ rad^2$ and hence

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$$P_{3N}(q=1/\ell_0) \approx \frac{(4-\beta)\,(\beta-1)\,\ell_0{}^4}{b_{coh}{}^2\, 2\pi^2\, r_e{}^2\, \lambda^2\, z} \tag{8}$$

In the case under consideration, with $\ell_0 \sim 10^8$ m, equation (7) predicts that the angular broadening points (the observations at the smallest scales: $r \sim 0.1 b_{coh}$) arc consistent with $P_{3N}(q \approx 10^{-8} m^{-1}) \sim 10^{25}$ m^{-3}, in approximate agreement with the centimeter wavelength scintillation index point. Thus the decorrelation bandwidth and angular broadening data do not rule out ℓ_0 as large as 10^8 m. (Even larger inner scales have been suggested to explain some of the refractive scintillation index data. The RISS data bearing on this issue arc presented in Figure 5 of Gupta *et al.* (1993). If m_{RISS} for nearby pulsars is examined, an inner scale of 10^7 to 10^9 m is suggested. Colts *et al.* (1987) interpret their m_{RISS} data in terms of ℓ_0 as large as 10^9 meters for nearby pulsars and smaller than about 10^7 m for more distant pulsars. Kaspi and Stinebring (1992), however, obtain values of m_{RISS} that require ℓ_0 smaller than 10^7 m.)

Three other observations argue that any inner scale cannot not be significantly larger than the $\sim 10^8$ m limit set by the DISS scintillation index. First, if there were inner scale substantially larger than 10^8 m, the structure function logarithmic slope at the coherence scale should be close to 2. The shape of DISS autocorrelation in radio frequency for a few high-SNR pulsars indicates α in the range 1.6- 1.9 and excludes α = 2 near the coherence scale for those lines-of-sight (Armstrong and Rickett 1981). These values of α arc what would be expected for either a pure, nearly-Kolmogorov power law near $q = 1/b_{coh}$ or a Kolmogorov power law with $\ell_0 \lesssim 10^8$ m. Thus the shape of the structure function near b_{coh} indicates that the inner scale cannot be substantially larger than 10^8 m. Second, the wavelength scaling of decorrelation bandwidth for PSR 0329+54 over an octave range of radio frequency (Cordes *et al.* 1985), another measure of structure function shape, is also consistent with α <2 near b_{coh}. (Cordes *et al.* 1985 measured the wavelength scaling of several pulsars, concluding that the data were consistent with a Kolmogorov spectrum, but only 0329+54 is close enough to be considered "local"). Third, VLBI measurements of the visibility function of radio sources that exhibit interstellar angular broadening suggest very small inner scales. Such VLBI observations have the potential to determine directly D_ϕ and its exponent α. Spangler and Gwinn (1990) assembled the relevant published data and concluded that the inner scale is in the range 0.5 -2.0 x 10^5 m. The radio sources in their study all exhibit enhanced scattering compared with that occurring within 1 kpc. Thus the small inner scales really have only been shown to apply for the regions of enhanced turbulence that are concentrated toward the inner galaxy or in the Cygnus region. Only two heavily scattered sources show the exponent α near *2.0;* the other 4 sources show α in the range $3.7 \pm 0.$ i 5. These results arc thus seriously inconsistent with the larger inner scales proposed by Gupta *et al.* (1993). (There is a technical question concerning the usc of self-calibration in the VLBI processing, since it removes the effect of image wander and so could bias estimates of a. However, the influence of self-calibration on estimates of α has yet to be determined, and is beyond the scope of this paper.) We note that Gupta et *al.* (1993) have suggested that a single inner scale may not apply for all the different regions of the ISM and that regions of enhanced turbulence may have smaller inner scales than does the local ISM.

We conclude that the data summarized here rule out ℓ_0 significantly larger than about 10^8 m and arc consistent with an inner scale much smaller than this.

Homogeneity of the ISM Electron Density

Finally, implicit in all of the above is the assumption that it is meaningful to combine data from different lines of sight to produce a composite spectrum. That is, wc have assumed that the ISM turbulence is homogeneous enough that there is a diffuse component having a unique spectrum. It is clear that even the local ISM is not strictly homogeneous (in time or space), as evidenced by the usc of terms like "enhanced", "depleted", and "extreme scattering events" to describe some observations. In addition to nonstationarity of the spectrum, there is the question of whether the density should even be viewed as a random process; the interpretation of a scattering observation as due to a medium having either a "statistical" or a "deterministic" description is to some extent subjective. (Composite spectral models arc possible--probably necessary--to explain some observations in detail. For example the description of the occasional focusing and double imaging conditions which appear in pulsar dynamic spectra (e.g. Hewish, Wolszczan and Graham 1985; Wolszczan and Cordes 1987; Cordes *et al.* 1993) and the rare but notable discrete propagation events (Fiedler *et al* 1987) arc probably best modeled in nonstatistical terms.)

However the observations summarized here indicate that, to within a band perhaps 2 orders of magnitude wide but extending over tens of orders of magnitude in spectral power, the data organize well about a unique statistical spectrum model. Such two-order-of-magnitude variations in spectral level arc also observed in the solar wind plasma density (e.g. Woo and Armstrong 1979). In the interplanetary medium it seems clear that the density is well-modeled as a ubiquitous random field with superimposed discrete structures, i.e. it does have a spectrum. We view the two order of magnitude variations in the inferred lSM spectral level as due partially to the simplifying assumptions used to interpret the observations (e. g., isotropic spectrum) and thus "error", and partially due to real ISM statistical inhomogeneity in time and space, and thus "ISM weather". While our conclusions regarding "the" ISM spectrum should be considered within the context of plus-or-minus an order of magnitude variations in spectral level, it should also be considered that comparable spectral inhomogeneity is observed in geophysical and other astrophysical fluids.

V. NATURE OF THE INTERSTELLAR PLASMA DENSITY SPECTRUM

in this section wc consider the nature of the spectrum shown in Figure 1, i.e. comment on how it might be produced, and what it implies for our understanding of interstellar turbulence. Nothing in this section will be original. We will simply cite relevant work on partially compressible plasma turbulence, and attempt to identify the points of correspondence with the observations in Figure 1. These comments will rely heavily on observational studies of the solar wind, an accessible turbulent fluid which has had extensive *in situ* measurements, and the theory which has addressed solar wind plasma turbulence.

The equations of magnetohydrodynamics show that fluctuations in plasma turbulence velocity, magnetic field, and density arc coupled. The $\beta=11/3$ interpretation of the data in Figure 1 therefore also strongly suggest that there arc accompanying fluctuations in magnetic field and plasma velocity on the same spatial scales. in fact, analogs drawn with the solar wind would indicate that the magnetic field and velocity fluctuations arc the *dominant* ones, with the density fluctuations being a dynamically unimportant tracer of the main turbulent energies. Thus a major goal of interstellar scintillation studies is to infer the properties of the invisible, yet dynamically important magnetic field and velocity fluctuations, given measurements of the detectable, yet dynamically unimportant density fluctuations.

Plasma turbulence may be thought of as either a superposition of large amplitude plasma waves, or as a less ordered state similar in nature to hydrodynamic turbulence. While it is unclear exactly where the transition occurs between these two types of turbulence, the extreme states may be found in the solar wind. The environments of shock waves, characterized by unstable ion distributions, host large amplitude MHD waves which arc readily identifiable with the wave modes of plasma physics textbooks. in the general solar wind far from shocks or comets, on the other hand, the situation more closely resembles that of fluid turbulence, without the presence of identifiable wave trains, and in which the spatial power spectra arc power laws over several decades of wavenumber. An excellent recent review of solar wind MHD turbulence is given by Marsch (1991). Both "wave turbulence" and "hydroturbulence" have associated density fluctuations.

A powerfu: inducement for considering solar wind turbulence as a model for interstellar turbulence is its possession of a Kolmogorov $q^{-11/3}$ density spectrum. Such a spectrum is repeatedly obtained over several decades of wavenumber in solar wind radio propagation studies (e.g. Woo and Armstrong 1979), and is observed much of the time in direct *in situ* spacecraft measurements. A recent discussion of this is given by Marsch and Tu (199(1), who report that Kolmogorov spectra for both magnetic field and density arc observed in the slow solar wind. The high speed solar wind also has an approximately Kolmogorov spectrum, but the cross helicity shows the "turbulence" to be propagating out from the Sun (in the rest frame of the wind). Like the solar wind, the data in Figure 1 exhibit a $q^{-11/3}$ spectrum over many decades. Its wide bandwidth makes it impossible to identify any specific wave processes. This is in contrast to the narrow band spectra seen near the Earth's bowshock and in computer simulations of collisionless shocks.

The wideband power law spectra found in the solar wind have stimulated several theoretical analyses of turbulence in plasmas. Of particular relevance here is that of Montgomery, Brown, and Matthaeus (1987--hereafter MBM). They provided a theoretical basis for density fluctuations in MHD turbulence, which was further elaborated by Shebalin and Montgomery (1988). The theory is an MHD equivalent of the famous Lighthill theory for the generation of sound by fluid turbulence. Briefly, incompressible MHD turbulence will have second order pressure fluctuations duc to changes in the fluid “ram pressure” and the magnetic energy density. These pressure fluctuations act as the source term for acoustic fluctuations. MBM considered the slow MHD fluctuations and found a solution with non-propagating (quasi-static) density fluctuations. They rcfcrrcd to these as "pseudosound", and wc note that they appear to bc equivalent to ponderomotive density fluctuations in the parlance of wave pictures of MHD turbulence.

Subject to a number of simplifying assumptions, MBM found that the pseudosound density spectrum had contributions from pressure fluctuations duc to kinetic energy and magnetic energy density variations. The problem of understanding the spectrum in $\delta\rho/\rho_0$ is reduced to that of understanding the spectra of δv and δB. Assuming $q^{-11/3}$ spectra for both, as observed in the solar wind, the resulting pseudosound spectrum has terms proportional to $q^{-13/3}$ (omnidirectional) duc to both magnetic and kinetic energy terms, and a term proportional to $q^{-11/3}$ produced by variations in the magnetic energy density. They pointed out that at sufficiently large q the - 11/3 contribution would dominate, in agreement with observations in the solar wind. Conversely, at low wavenumbers the steeper component should dominate -- a result that persists even in the more general expressions of Matthaeus *et al.* (199 1). However, in neither the solar wind nor in the data of Figure 1 dots such a low wavenumber steepening appear. The pseudosound theory was expanded upon by Matthaeus *et al. (1991),* who also analyzed *Helios* spacecraft observations in a search for observational evidence that the density fluctuations were pseudosound in nature. The observational signature searched for was a dependence $\delta\rho \propto (\delta B)^2$, which is a defining characteristic of ponderomotive density fluctuations. They found only fair agreement, in that the $\delta\rho$ exhibited terms proportional to δB as well as to $(\delta B)^2$. Wc note here that Zank and Matthaeus (1992b) have reconsidered the possibility of first order density fluctuations.

Among the assumptions in the pseudosound theory, the isotropy of the turbulence is the one most subject to suspicion. The imposition of a strong magnetic field on a plasma renders its turbulence two dimensional (Montgomery and Turner 1981; Zank and Matthaeus 1992a) in the sense that the magnetic and velocity fluctuations lie in planes perpendicular to the large scale field, and that gradients along the field arc much smaller than perpendicular to it. Thus when a strong magnetic field is prc.sent, the turbulence is *highly* anisotropic. This concept is strongly supported by the observations of field-a]igncd anisotropic density fluctuations in the inner solar wind (i.e. Armstrong *et al.* 199(1). Obviously in the case of a high β plasma or one in which the static field is small compared with the fluctuations, the turbulence can be isotropic, but it is unclear at the present time whether this applies in the lSM.

An alternative suggestion for the production of density fluctuations in interstellar MHD turbulence was given by Higdon (1984, 1986). Higdon realized that the theoretical results on the two dimensional nature of MHD turbulence with a strong magnetic field had potential significance for the ISM. Hc also made an exhaustive survey of the state of knowledge regarding density fluctuations in hydrodynamic turbulence, on the sensible assumption that turbulent MHD fluids would have closely similar compressive mechanisms. The papers of Higdon made two important theoretical contributions. The first was that the heating rate of the interstellar medium duc to the damping of turbulence could be much less than that suggested in other calculations, in which isotropic turbulence was assumed. The second was the suggestion that the major contribution to the density fluctuations in interstellar turbulence was from "nonpropagating entropy structures" rather than MHD waves. In distinction with the results of MBM, Higdon (1984, 1986) dismissed Lighthill radiation because of its steep spectrum. However, this objection is removed by the result of MBM that MHD Lighthill radiation has a flatter spectrum than dots I ID Lighthill radiation. A major attraction of the pseudosound theory has been its ability to explain the form of the density spectrum, as similar to the spectrum of the velocity and magnetic field. However, the recent work of Bayly, Levermore and Passot (1992) indicates that the "Kolmogorov" density spectrum is considerably more general. invoking non-wave-]ikc fluctuations in fluids and discussing extensions to MHD fluids, they conclude that "it appears that a $k^{-5/3}$ inertial range density spectrum can be the consequence of purely entropic (or thermal) effects, purely magnetic effects or a combination thereof". Thus a host of turbulence models might be capable of reproducing our primary observational result in Figure 1.

The foregoing discussion has emphasized the lessons learned from the solar wind, as a relatively accessible turbulent plasma. Wc now comment on the consequences of the spectrum in Figure 1 for plasma processes in the lSM. I'here are regions of the ISM with markedly different densities and temperatures. It cannot be considered as a single statistically homogeneous medium, rather as a number of discrete "phases" that may locally support homogeneous turbulence.

In a recent study Spangler (1991) considered what region is responsible for the localized enhanced plasma turbulence, that causes very heavy radio wave scattering for some lines of sight in the Galactic plane. He concluded that the most likely regions were the extended envelopes of HII regions and the warm ionized medium (WIM) of the McKee and Ostriker (1977) model. In his Section IV he then asked whether the cooling capacity of these regions could accommodate the power dissipated from the turbulence. His equation (38) gives the radiative cooling rate as proportional to the square of the mean electron density in a region, which for the regions in question is relatively high (0.2 to 4 cm^{-3}). His Figure 3 shows that they could accommodate the expected heating, if the outer scale of the turbulence were larger than about 10^{13} m, which is comfortably smaller than tthc typical size of these regions.

Even though the local turbulence, parameterized by C_N^2, is relatively weak, the cooling rate capacity is much less than in the regions of enhanced turbulence, because the mean density is much less. The mean number density from pulsar dispersions is about 0.025/f cm^{-3}, where the filling factor f is probably not much less than 1.0 (see Taylor and Cordes 1993). The turbulent power to be dissipated cannot be determined from the density spectrum. We estimate it as the turbulent energy density divided by a characteristic time; the former we estimate as $\delta B^2/(8\pi)$; the latter we estimate as the outer scale divided by the Alfven speed. We further assume $\delta B \approx gB_0$, with g less than or near one and the mean field B_0 is, say, 3μG. Heiles (1987) suggests that this figure is similar in both dense and diffuse HII regions. The resulting turbulent power is approximately $g^2 B_0^3/[8\pi(4\pi\rho)^{0.5} L_0]$. In order that the dissipation of this power lies below the cooling capacity of the diffuse HII medium, the outer scale L_0 must be greater than about 50 pc, which is not much smaller than the expected size of such regions. For a smaller outer scale, the turbulence ratio g and/or the filling factor f must be correspondingly less than 1.0. Spangler and Reynolds (1990) drew a similar conclusion about the local diffuse HII medium, in their comparison of Hα and radio scattering measurements.

A second interesting consequence from Figure 1 is the implication that turbulence exists at and beyond the scale of ion-neutral collisions. Discussions of turbulence in molecular clouds and star forming regions attribute considerable importance to the critical scale at which the frequency of an Alfven wave equals the ion-neutral collision frequency (e.g. Mouschovius 1991; McKee *et al.*

1993). The theoretical basis of these discussions is McIvor's (1 977) description of the propagation of Alfven waves in a partial] y ionized medium. The theory predicts that when the wave frequency (ω) equals the collision frequency (ν_{in}), the waves become evanescent. For $\omega < \nu_{in}$ the neutral gas participates in the wave motion, whereas for $\omega > \nu_{in}$, only the ions participate in the wave motion, which is partially damped. Whereas a careful examination of the theory shows that waves continue to exist when $\omega >> \nu_{in}$ the existence of the critical collisional scale appears to have been interpreted in some papers as an absolute minimum scale for the turbulence.

In the partially ionized interstellar plasma the dominant ion collisions are with either neutral hydrogen or helium (Spangler 1991), which may be the major heating mechanism for the neutral gas. The corresponding collisions] scale V_A/ν_{in} may be calculated to be 10^{13} to 10^{14} m. The spectrum in Figure 1 extends to wavenumbers several decades above 10^{-14} m^{-1}, implying that density and presumably magnetic irregularities extend to scales much smaller than the collisional scale. Significantly, there is no sign of a break or other spectral feature near the collisional scale, at which waves should apparently not exist. We conclude that Figure 1 may indicate that the linear theory of Alfven waves cannot be used to place strong constraints on turbulent irregularities in a partially ionized plasma.

VI. SUMMARY AND CONCLUSIONS

Radiowave scattering data taken on local ($\lesssim$ 1 kpc) lines-of-sight in the ISM have been summarized in terms of the phase structure function (Figure 2). Where possible, we also summarize the structure function's local logarithmic slope, α, for each observation. The scale size range for the radio data in Figure 2 is $10^{6.3}$ to 10^{13} m. The available data are consistent with $\alpha < 2$ over this range, allowing us to connect wavenumbers with spatial scales and infer a composite power spectrum for the electron density in the local ISM. The data are most simply interpreted in terms of a pure powerlaw having a nearly Kolmogorov spectrum, over wavenumbers ~10^{-13} to $10^{-6.3}$ m^{-1}. Thus the outer scale, as determined from the radio data alone, must be larger than or of order 10^{13} m. The radio data are consistent with an inner scale smaller than or of order 2 X 106 m, with an upper limit to the inner scale of - 10^8 m. When the radio observations are combined with even lower wavenumber data based on gradient scale of electron density, interstellar cloud density, and data on ISM velocity fluctuations, the aggregate spectrum set is approximately power law spectrum between $q \sim 10^{-16}\,m^{-1}$ and $10^{-6}\,m^{-1}$. The combined data set organizes remarkably well about a Kolmogorov model $P_{3N} = 10^{-3}\,m^{-6.67}\,q^{-11/3}$ over this huge wavenumber range, with values of the spectral index, β, greater than 4 and less than about 3.5 ruled out. The possibility that β equals 4 is a special case that corresponds to scattering by discontinuities in the plasma density; the data arc just compatible with (his model over the range $\sim 10^{-13}$ to $10^{-6}\,m^{-1}$ if an outer scale $\sim 10^{14}$ to 10^{16} m is invoked.

In Section V, wc noted the advances stimulated by the existence of density fluctuations with a similar spectrum in the solar wind. in the solar wind there is also evidence for similar spectra in the magnetic field and plasma velocity. A particularly appealing theoretical description of density fluctuations is that they arc "pseudosound" fluctuations. Pressure fluctuations, due to spatial and temporal variations of magnetic field and plasma velocity, drive small amplitude density fluctuations. This theory explains the $q^{-11/3}$ density spectrum seen in both the solar wind and the ISM. However, recent theoretical investigations of plasma turbulence also suggest that pressure balance structures could also produce such a spectrum. The resolution of the nature of the fluctuations in the lSM may ultimately require information comparable to that available for the solar wind, on magnetic and kinetic energy characteristics.

Assuming that the density spectrum gives evidence for Alfvenic turbulence in the ISM, wc asked whether the power dissipated by the turbulence could exceed the cooling capacity of its low density host plasma. If the outer scale of the turbulence is as large as 30 pc, the dissipation can be accommodated by radiative cooling. The most likely dissipation process involves ion-neutral collisions which have a critical scale the "Alfven-collisional scale". There is no unusual signature in our measured spectrum near the corresponding wavenumber. This may imply that Alfven wave properties arc not applicable to fully developed turbulence in a partially ionized plasma.

APPI:NDIX

ISM TURBULENCE SPECTRUM AND '1'111; SEARCH FOR A BACKGROUND OF VERY-LOW-FREQUENCY GRAVITATIONAL WAVES

Gravitational wave searches arc conventionally classified as higl~-frequency (10 to 10000 Hz), low-frequency (10^{-5} to 10 Hz), or very low frequency (frequencies lower than 10^{-5} Hz) (Thorne 1987). These bands divide along detector techniques: resonant bars and laser interferometers (high frequency); Doppler tracking of spacecraft, normal modes of the earth and sun, beam detectors in space (low-frequency); pulsar timing and the timing of orbital motions (very-low-frequency). In searches for a very-low-frequency stochastic gravitational wave background, the earth and a pulsar act as free test masses. The relative velocity of the earth and the pulsar is perturbed by buffeting of the earth and pulsar by gravitational radiation (e.g. Estabrook and Wahlquist 1975; Bertotti, Carr and Rees 1983; Hellings and Downs 1983; Romani and Taylor 1983; Stinebring *et al.* 1990). This buffeting of source and observer by gravitational waves will produce time-of-arrival (TOA) fluctuations. Observed TOA fluctuations include both gravitational wave perturbations (at some level) and various "noises". Prior to the discovery of PSR 1937+21, upper limits to a gravitational wave background in the very-low-frequency band were apparently set by intrinsic timing noise of the pulsars being observed. PSR 1937+21, however, has very low intrinsic timing noise and observations may ultimately be limited by RISS (Armstrong 1984). In this Appendix, wc calculate the contribution to pulsar timing noise caused by propagation through interstellar turbulence having the spectrum given in Figure 1 and wc compare this noise level with TOA observations at low Fourier frequencies.

Near the wavenumbers relevant for very-low-frcclucncy gravitational wave searches, the spectrum in Figure 1 is approximately $C_N^2\, q^{1\ 1/3}\ m^{-3}$, with $C_N^2 \approx 10^{-3}\ m^{-6.67}$. An electromagnetic wave propagating through homogeneous turbulence having such a spectrum develops phase perturbations. in the observer's plane, these phase fluctuations arc a function of (x,y), the coordinates perpendicular to the line of sight to the source. The two dimensional spectrum of phase in the observer's plane, $P_{2\phi}(\kappa)$, is $P_{2\phi}(\kappa) = 2\pi z \lambda^2 r_e^2 C_N^2\ K\text{-l}^{\ 1/3}$, where z is the propagation distance. Assuming that the turbulence is frozen and that the pattern sweeps across the observer's plane with some velocity, v_{pat}, this spatial spectrum is converted into a mlc-dimensional temporal spectrum of phase (see, e.g., Armstrong 1984 for a discussion in the RISS context): $P_{1\phi}(f) = 2^{-2/3}\pi^{-1/6}[\Gamma(4/3)/\Gamma(11/6)]\, z\, \lambda^2\, r_e^2\, C_N^2\, v_{pat}^{5/3}\, f^{-8/3}$. Since frequency is the derivative of phase, this can be restated in terms of the spectrum of fractional frequency fluctuations, $S_y(f)$, where $y(t) = \Delta\nu/\nu_0$, and ν_0 is the center frequency: $S_y(f) = 2^{-2/3}\pi^{-1/6}[\Gamma(4/3)/\Gamma(11/6)]\, z\, c^{-2}\lambda^4 r_e^2\, C_N^2\, v_{pat}^{5/3} f^{-2/3}$. Fractional frequency fluctuations arc the time derivative of the pulsar timing residuals (Hellings and Downs 1983), so that their spectra are related by $S_m(f) = S_y(f)/[4\pi^2 f^2]$, where S_m is the spectrum of TOA residuals. $S_y(f)$ is related to the Al Ian variance, $\sigma_y^2(\tau)$, of a time series according to $\sigma_y^2(\tau) = 4\int_0^\infty S_y(f)\sin^4(\pi\tau f)/(\pi\tau f)^2\, df$ (Barnes *et al.* 1971).

Stinebring *et al.* (1990) show power spectra of timing residuals for 1937+21 at λ =0. 126 m, achieving lowest spectral density (expressed here as S_y) of 4.5 (+5.5, -2.0) X 10^{-20} Hz^{-1} at f = 1.2 X 10^{-8} Hz. Taking z = 3.58 kpc (Taylor, Manchester, and Lyric 1993), $C_N^2 = 10^{-3} m^{-6.67}$ and v_{pat} = 100 kn~/see, the predicted S_y(1.2 X 10^{-8} Hz) is 5 X 10^{-20} Hz^{-1}; thus the data arc at or near the RISS-limit at this frequency. (The associated square-root of Allan variance predicted by the spectrum in Figure 1 is $\sigma_y(\tau)$ = 4 X 10^{-14} $(\tau/1\ \text{year})^{-1/6}$, comparable on time scales of order years to the estimated stabilities of the best atomic clocks.)

This RISS limit for timing of 1937+21 can be restated in tcms of a limit on the energy density of a gravitational radiation background. It is conventional (Detweiler 1979) to assume that the gravitational wave background spectrum is white. over an octave bandwidth centered on some Fourier frequency (f_c) and then to relate the variance of the TOA residuals (σ^2) to the density of gravitational waves (p) in that band that would produce that observed TOA variations: p = 243 $\pi^3 f_c^4 \sigma^2$/ 208 G. Here G is the Newtonian gravitational constant. (The critical density required to close the universe gravitationally, p_c, is $3H_0^2/8\pi G \approx$ 5.5 X 10^{-30} grams/cm^3 if H_0 = 55 km/sec Mpc^{-1}.) The spectrum of Figure 1 predicts that 1937+21's RISS-generated TOA fluctuations at λ = 0.126 m correspond to $\rho(f_c)/\rho_c \approx$ 4.5 X 10-7 $(f_c/10^{-8}\ Hz)^{7/3}$. This bound is comparable to that dervied by Stinebring *et al.* (1990) at f_c= 4.4 X 10^{-9} Hz. Improvements to sensitivities better than this will require observations at higher frequencies or dual frequency observations to calibrate and remove RISS noise.

ACKNOWLEDGMENTS

Wc thank W. A. Colts and R. Woo for discussions on interstellar and interplanetary scintillation. JWA's contribution to this work was supported by the Ultraviolet/Visible/Gravitational Physics Branch of the NASA Astrophysics Division and performed at the Jet Propulsion Laboratory, California institute of Technology, under contract with NASA. SRS acknowledges support from grant NAGW- 1594 from NASA and ATM 92-16821 from the NSF.

REFERENCES

Armstrong, J. W. 1984, *Nature* 307,527

Armstrong, J. W. and Rickett, B. J. 1981, *MNRAS* 194,623

Armstrong, J. W., Cordes, J. M., and Rickett, B. J. 1981, *Nature*, **291**, 561

Armstrong, J. W., Coles, W. A., Kojima, M. and Rickett, B. J. 1990 *Ap. J.*, **358**, 686

Backer, D. C. 1975, *Astron. Astrophys.* **43**, 395

Barnes, J. A. *et al.* 1971, *IEEE Trans.*, **IM-20**, 105

Bayly, B. J., Levermore, C.D. and Passot, T. 1992, *Phys. Fluids, 4,945*

Bertotti, B., Carr, B. J., and Rees, M. J. 1983, *MNRAS*, 203, 945

Blandford, R. and Narayan, R. 1985, *MNRAS, 213, 591*

Blandford, R., Narayan, R. and Romani, R. W. 1986, *Ap. J.*, 301, 53

Caplan, S. A. 1966, *Interstellar Gas Dynamics* (Oxford: Pergammon)

Cesarsky, C. J. 1980, *ARAA*. 18, 289

Clegg, A. W., Cordes, J. M., Simonetti, J. H., and Kulkarni, S. R. 1992, *Ap. J.*, 386, 143

Cognard, 1., Bourgois, G., Lestrade, J-F, Biraud, F., Aubry, D., Darchy, B., and Drouhin, J-P 1993, *Nature, 366320*

Cohen, M. H. and Cronyn, W. M. 1974, *Ap. J.*, 192, 193

Colts, W. A., Frehlich, R. G., Rickett, B. J. and Codona, J. 1..1987, *Ap. J.*, 315, 666

Cordes, J. M. 1986, *Ap.* 3.,311, 183

Cordes, J. M. and Helfand, D. J. 1980, *Ap. J.*, 239, 640

Cordes, J. M., Pidwerbetsky, A., and Lovelace, R. V. E. 1986, *Ap. 1.,310,* 737

Cordes, J. M. Weisberg, J. M., and Boriakoff, V. 1985, *Ap. J.* 288, 221

Cordes, J. M. Weisberg, J. M., Frail, D. A., Spangler, S. R. and Ryan, M. 1991, *Nature, 354, 121*

Cordes, J. M., Wolszczan, A., Dewey, R, J., Blaskiewicz, M., and Stinebring, D. R. 1990, *Ap. J., 349, 245*

Cornwell, T. J. and Narayan, R. 1993, *Ap. J.Lett.*, 408, L69

Dashen, R. and Wang, G-Y 1993, *JOSA*, A1O, 1219.

Dennison, B., Fiedler, R. and Johnston, K. J. 1987, *Ap. J.*, 313, 141

Detweiler, S. 1979, *Ap J., 234,* 1100

Duffett-Smith, P. J. and Readhead, A. C. S. 1976, *MNRAS*, 174, 7

Fey, A. L., Spangler, S. R., and Mutel, R. L. 1989, *Ap. J.*, 337, 730

Fiedler, R. L., Dennison, B., Johns[on, K. J., and Hewish, A. 1987, *Nature,* 326, 675

Gehrels, N and Chen, W. 1993 Nature, 361, 706

Goodman, J. and Narayan, R. 1985, *MNRAS, 214,* 519

Gupta, Y., Rickett, B. J., and Colts, W. A. 1993, *Ap. J.* **403**, 183

Gwinn, C. R., Bartel, N. and Cordes, J. M. 1993, *Ap. J.*, 410, 673

Hajivassiliou, C. and Duffett-Smith, P. 1990, *MNRAS, 247,* 491

Harrison, P. A., Lyne, A. G. and Anderson, B. 1993, *MNRAS*, 261, 113

Heiles, C. 1987 in *Interstellar Processes* ed. P. J. Hollenbach and H.A. Thronson (Dordrecht: Reidel) p 171

Hewish, A. 1980, *MNRAS 192, 799*

Hewish, A., Wolszczan, A., and Graham, D. A. 1985, *MNRAS*, 213, 167

Higdon, J. C. 1984, *Ap. J.*, 285, 109

Higdon, J. C. 1986, *Ap. J.*, 309, 1342

Ishimaru, A. 1978, *Wave Propagation and Scattering in Random Media,* Vol. 2, p.538 (New York: Academic Press)

Jokipii, J. R. 1974, *Interaction of Charged Particles with Random Electromagnetic Fields in Astrophysics* (summer school on Plasma Instabilities and Turbulence, Aix-en-Provence)

Jokipii, J. R. 1988, in *Radio Wave Scattering in the Interstellar Medium* (AIP Conf. Proc. 174), ed. J. M. Cordes, B. J. Rickett, and D. C. Backer, p. 48 (New York: American institute of Physics)

Kaspi, V. A. and Stinebring, D. R. 1992, *Ap. J.* 392, 530

Kaplan, S. A. 1966, *interstellar Gas Dynamics* (Oxford: Pergamon Press)

Kraichnan, R. H. 1965, *Phys. Fluids, 8,* 1385

Larson, R. B. 1979, *MNRAS, 186, 479*

Lazio, T., J., Spangler, S. R., and Cordes, J. M. 1990, *Ap. J.* 363, 515

Leckband, J. A. 1992, Ph.D. Thesis, University of lows.

Lee, L. C. and Jokipii, J. R. 1977, *Ap. J. 218,* 148

Lovelace, R. V. E. 1970, Ph. D. thesis, Cornell University

Marsch, E. 1991, "MI ID Turbulence in the Solar Wind", in *Physics of the Inner Heliosphere 2,* R. Schwenn and E. Marsch, editors, (Springer Verlag: Berlin).

Marsch, E. and Tu, C. Y. 1990, *J. Geophys. Res*, **95**, 11945

Montgomery, D. and Turner, L. 1981, *Phys. Fluids, 24,* 825

Montgomery, D., Brown, M. R., and Matthaeus, W. H. 1987, *J. Geophys. Res., 92, 282.*

Matthaeus, W. H., Klein, L. W., Ghosh, S., and Brown, M. R. 1991, *J. Geophys. Res.*, 96, 5421

McIvor, I., 1977, *MNRAS, 178,* 85

McKee, C. F. and Ostriker, J. P. 1977, *Ap. .7.*, 218, 148

McKee, C. F., Zweibel, E. G., Goodman, A. A. and Heiles, C., 1993, in *Protostars and Planets III*, cd. E. H. Levy and J. 1. Lunine, U. of Arizona Press

Mouschovias, T. C., 1991, *Ap. J.*, 373, 169

Narayan, R. 1992, *Phil. Trans. Royal* Sot., *341, 151* (1992)

Narayan, R. and Goodman, J. 1989, *MNRAS, 238, 963*

Neugebauer, M., Wu, C. S., and Hubs, J. D. 1978, *J. Geophys. Res.* **83**, *1027*

Ovendon, C. R., Shah, H.A. and Schwartz, S. J. 1983, *J. Geophys. Res.*, **88**, 6095

Phillips, J. A. and Clegg, A. W. 1992, *Nature 360, 137*

Phillips, J. A. and Wolszczan, A. 1991, *Ap. J.* 382, L27

Readhead, A. C. S. and Hewish, A. 1974, *MNRAS, 78, 1*

Readhead, A. C. S. and Duffett-Smith, P. J. 1975, *Astron. Astrophys., 42, 151*

Resch, G. M. 1974, Ph. D. thesis, Florida State University

Rickett, B. J. 1970, *MNRAS, 150, 67*

Rickett, B. J. 1977, *Ann. Rev. Astr. Ap.* 1S, 479 [R77]

Rickett, B. J. 1986, *Ap. J.*, 307, 564

Rickett, B. J. 1988, in *Radio Wave Scattering in the Interstellar Medium* (AIP Conf. Proc. 174), cd. J. M. Cordes, B. J. Rickett, and D. C. Backer, p. 2 (New York: American Institute of Physics)

Rickett, B. J. 1990, *Ann. Rev. Astr. Ap. 28, 561* [R90]

Rickett, B. J., Colts, W. A., and Bourgois, G. 1984, *Astron. Astrophys., 143, 390*

Rickett, B. J., and Lyric, A. G. 1990, *MNRAS, 244,* 68

Romani, R. W., Narayan, R., and Blandford, R. 1986, *MNRAS, 220, 19*

Romani, R. W. and Taylor, J. H. 1983, *Ap. J.*, 26S, 1.35

Rumsey, V. H. 1975, *Radio Sci.*, 10, 107

Ryba, M. 1991, Ph. D. thesis, Princeton University

Salpeter, E. E. 1969, *Nature*, 221, 31

Scheuer, P. A. G. 1968, *Nature*, **218**, 920

Shebalin, J. V. and Montgomery, D. 1988, *J. Plasma Phys.*, **39**, 339

Shishov, V. .1. *1974, Soviet Astron.* **17**, 598

Simonetti, J. H. 1992, *Ap. J.*, **386**, 170

Simonetti, J. H. and Cordes, J. M. 1988, in *Radio Wave Scattering in the Interstellar Medium* (AIP Conf. Proc. 174), cd. J. M. Cordes, B. J. Rickett, and D. C. Backer, p. 134 (New York: American Institute of Physics)

Simonetti, J. H., Cordes, J. M., and Spangler, S. R. 1984, *Ap. J.*, 284, 126

Smith, F. G. and Wright, N. C. 1985, *MNRAS*, *214, 97*

Spangler, S. R. 1987, *Phys. F/., 30,* 1104

Spangler, S. R., Fey, A, L., and Cordes, J. M. 1987, *Ap. J.,* 322, 909

Spangler, S. R., Eastman, W. A., Gregorini, I.., Mantovani, F., L. and Padrielli, L,. 1993, *Astron. Astrophys*, *267,* 213

Spangler, S. R. and Reynolds, R. J., 1990, *Ap. J.*, 361, 116

Spangler, S. R. and Gwinn, C. R. 1990, *Ap. J. 3S3,* 1.29

Stinebring, D. R, Ryba, M. F., Taylor, J. H., and Romani, R. W. 1990, *Phys. Rev. Lett.*, 65, 285

Sutton, J. M. 1971, *MNRAS,* **155,** 51

Taylor, J. H. 1991, *Proc. IEEE, 79, 1054*

Taylor, J. H and Cordes, J. M. 1993, *Ap. 1.,411,* 674

Taylor, J. H., Manchester, R. N., and Lyne, A. G. 1993, *Ap. J. Supp.* 88, 529

Thorne, K. S. 1987 in 300 *Years of Gravitation* , *S.* W. Hawking and W. Israel eds., p. 330 (Cambridge: Cambridge University Press)

Vandenberg, N. R. 1974, Ph. D. thesis, University of Maryland

Wolszczan. A. and Cordes, J. M. 1987, *Ap. J. Lett.*, 320, 1,35

Woo, R. and Armstrong, J. W. 1979, *J. Geophys. Res.*, 84, 7288

Zank, G. P. and Matthaeus, W. H. 1992a, *J. Plasma Phys.*, 48, 85

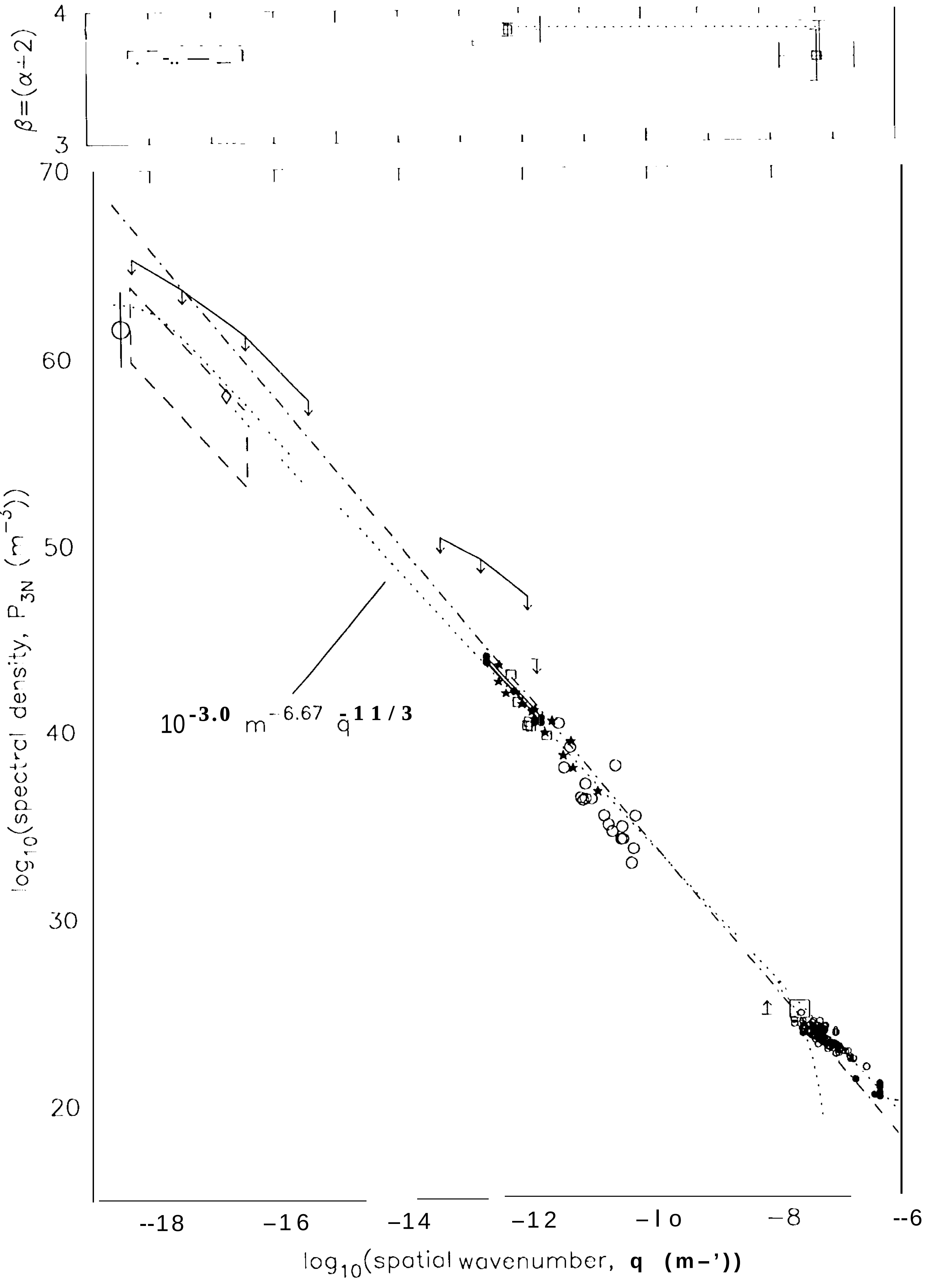
Zank, G. P. and Matthaeus, W. H. 1992b, in *Solar Wind 7,* E. Marsch and R. Schwenn, editors, Cospar Colloquium Series Vol 3, p 587

FIGURE CAPTIONS

Figure 1. Lower Panel: Logarithmic plot of inferred three-dimensional electron density power spectrum versus wavenumber. References for the observations and symbols for the technique involved arc given in the text; from high to low wavenumber the data are from angular broadening, coherence bandwidth, weak DISS at centimeter wavelengths, strong DISS at meter wavelengths (lower limit), refractive steering of DISS by large structures, RISS scintillation indices, DM fluctuations, pulsar timing (upper limits), rotation measure fluctuations (upper limits), velocity fluctuations, ISM cloud densities, and gradient scale. Dotted lines arc Kolmogorov spectra with outer scale = 10^{18} m and inner scales of 10^5 m and 10^8 meters. Dot-dashed line is model spectrum having spectral index of 4, normalized to pass through the cluster of points near (1 / coherence scale for meter wavelength observations). Upper panel: Estimates of the spectral index, β, as a function of wavenumber. Vertical line and box near 10-7 m^{-1} arc from shape of 1SS dynamic spectra and wavelength scaling of decorrelation bandwidth. Values near 10^{-12} m^{-1} arc from dispersion measure structure function. Dotted line is average spectral index between dispersion measure structure function data and the coherence scale points near 1()-7 m^{-1}. Dashed box is range of slopes from lSM velocity fluctuations data. Kolmogorov value is $\beta = 11/3$.

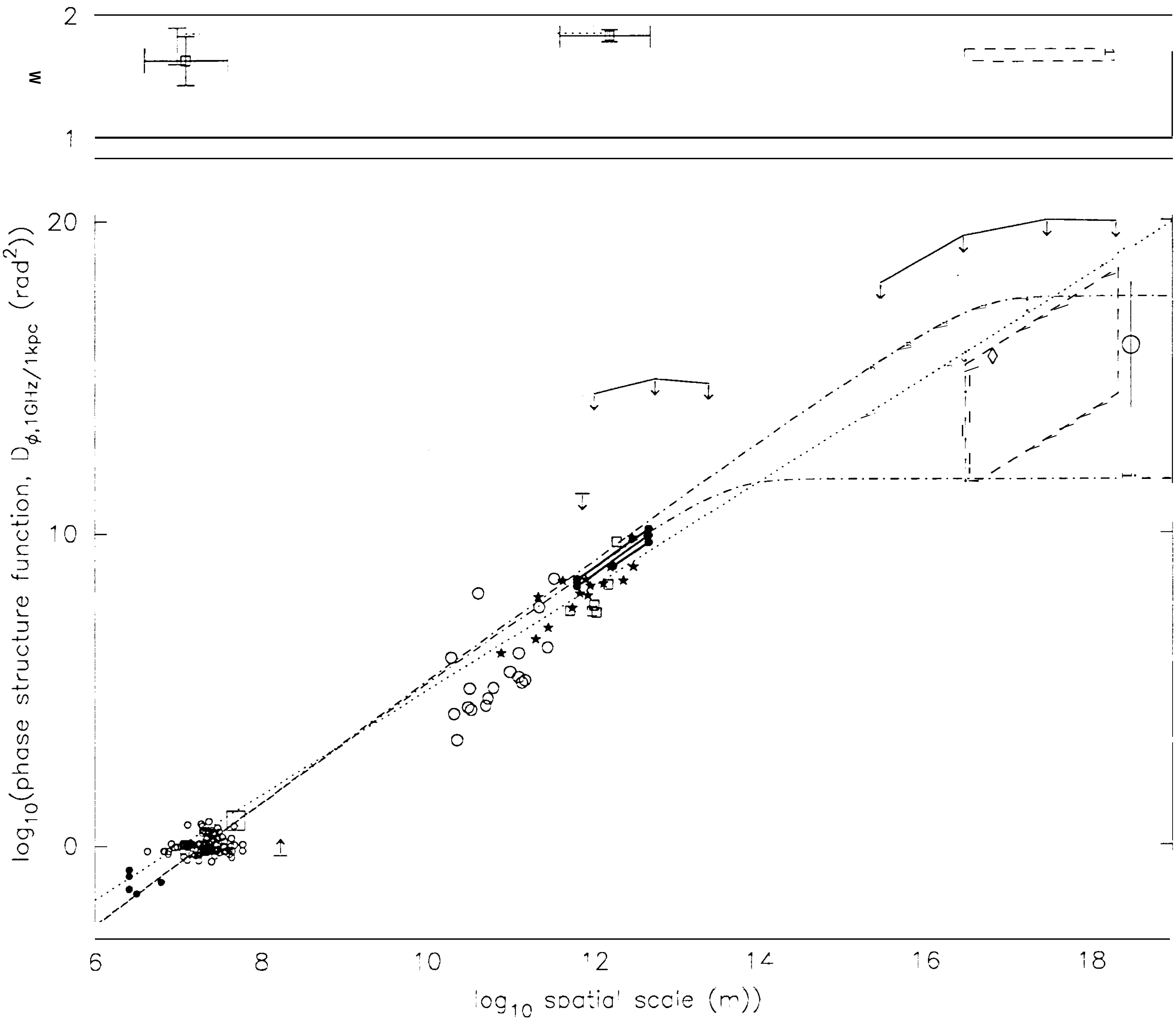
Figure *2.* Lower Panel: Logarithmic plot of inferred wave structure function for radio waves at 1 GHz and 1 kpc propagation distance; symbols are as in Figure 1. Dotted lines arc Kolmogorov spectra with outer scales of 10^{13} m and 10^{16} m. Dot-dashed line is model spectrum having spectral index of 4, normalized to pass through the cluster of points near the coherence scale for meter wavelength observations, having outer scales of s of 10^{13} m and 10^{16} m. Upper panel: logarithmic slope of structure function with baseline (symbols have same meaning as upper panel in Figure 1). If α <2, then $\beta = \alpha + 2$ (see text).

Figure 1.



$\beta=(\alpha-2)$
4
3
70
60
50
40
30
20
$\log_{10}$(spectral density, P_{3N} (m^{-3}))
$10^{-3.0}$ $m^{-6.67}$ $q^{-11/3}$
--18
-16
-14
-12
-10
-8
--6
$\log_{10}$(spatial wavenumber, q (m^{-1}))

Figure 2



β
2
1
20
10
0
6
8
10
12
14
16
18
log10(phase structure function, Dϕ,1GHz/1kpc (rad2))
log10 spatial scale (m))